Learning to be Smooth: An End-to-End Differentiable Particle Smoother

Ali Younis
ayounis@uci.edu

Erik Sudderth
sudderth@uci.edu

UCI University of California, Irvine

1. Introduction

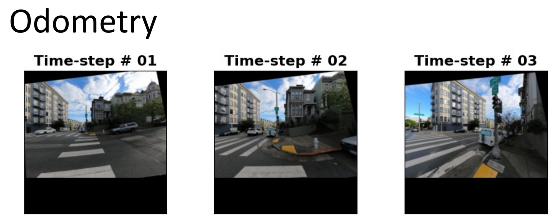
New discriminative learnable particle smoother method: Mixture Density Particle Smoother

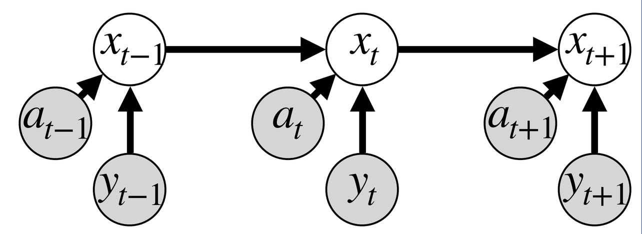
Given

x_t = state a_t = action y_t = observation

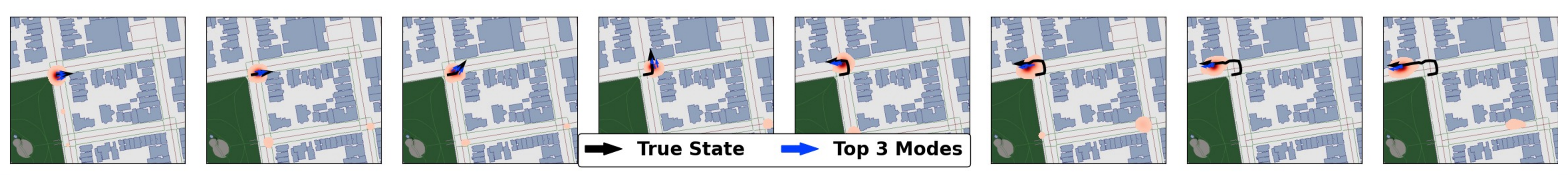
Actions *(optional)*: E.g. Noisy Odometry

Observations: E.g. Images





Estimate Posterior Density 3D state: x_t = (position, orientation)



2. Mixture Density Particle Filter (MDPF)

Step 0: Initialize Particle Set

$$\{x_1^{(1)},\ldots,x_1^{(N)}\} \quad \{w_1^{(1)},\ldots,w_1^{(N)}\}$$

(particles) *(weights)*

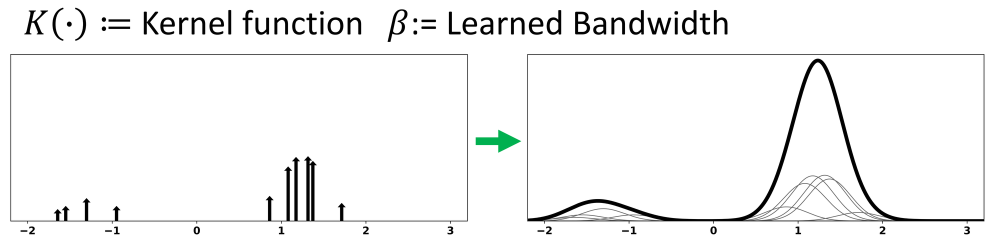
Step 1: (Differentiable) Particle Resampling

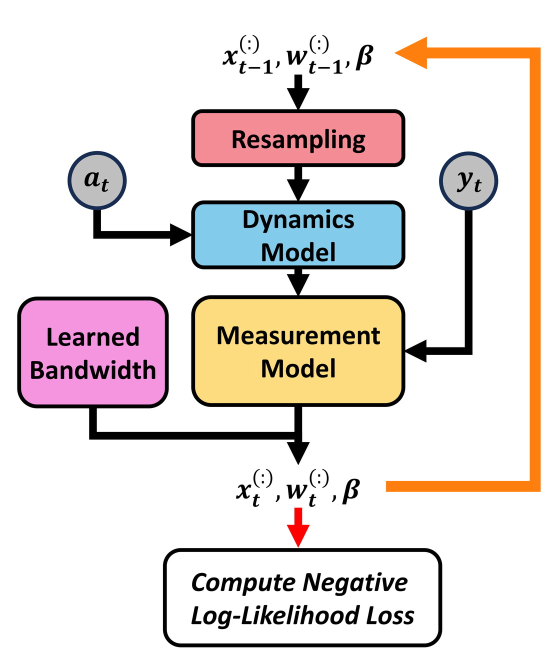
$$m(x|\phi) = \sum_{j=1}^{N} w_t^{(j)} \cdot K(x; x_t^{(j)}, \beta)$$

$$\tilde{x}_t^{(i)} \sim m(x|\phi) \qquad \phi = \{x_t^{(\cdot)}, w_t^{(\cdot)}, \beta\}$$

$$\tilde{w}_t^{(i)} = \frac{m(\tilde{x}_t^{(i)}|\phi_0)\big|_{\phi_0=\phi}}{m(\tilde{x}_t^{(i)}|\phi_0)} = 1 \qquad \nabla_\phi \tilde{w}_t^{(i)} = \frac{\nabla_\phi\, m(\tilde{x}_t^{(i)}|\phi_0)\big|_{\phi_0=\phi}}{m(\tilde{x}_t^{(i)}|\phi_0)}$$

$K(\cdot)$:= Kernel function β:= Learned Bandwidth





Step 2: Particle Proposal (Noisy Dynamics)

$$x_t^{(i)} \sim f(x_t | x_{t-1} = x_{t-1}^{(i)}, a_t, \eta) \qquad \eta \sim N(0,1)$$

Step 3: Measurement Update

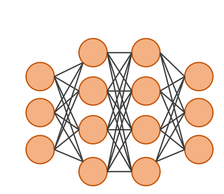
$$w_t^{(i)} = p(y_t|x_t^{(i)}) \cdot w_{t-1}^{(i)} \qquad w_t^{(i)} = l(x_t^{(i)}|y_t) \cdot w_{t-1}^{(i)}$$

(Generative) *(Discriminative)*

Normalize: $\sum_{i=1}^{N} w_t^{(i)} = 1$

Learnable Particle Filters:

$f(x_t|x_{t-1} = x_{t-1}^{(i)}, a_t, \eta)$, $l(x_t^{(i)}|y_t)$:=



Neural Network

Trained end-to-end

3. Existing Two Filter Smoother

To compute the smoothed posterior distribution:

$$p(x_t|y_{1:T}) = \frac{p(y_{t+1:T}|x_t)\, p(x_t|y_{1:t})}{p(y_{t+1:T}|y_{1:t})} \propto p(y_{t+1:T}|x_t) p(x_t|y_{1:t})$$

Observation Likelihood Forward Filtering

Introduce auxiliary distribution:

Backward Filtering

$$\bar{p}(x_t|y_{t+1:T}) = \frac{p(y_{t+1:T}|x_t)}{\gamma_t} \propto p(y_{t+1:T}|x_t)$$

(Assuming Bounded State Space)

Forward Filtering Backward Filtering

$$p(x_t|y_{1:T}) \propto p(x_t|y_{1:t}) \cdot \bar{p}(x_t|y_{t+1:T})$$

$$p(x_t|y_{1:t}) = \sum_{i=1}^{N} w_t^{(i)} \delta(x_t - x_t^{(i)})$$

Re-write to derive a particle based algorithm:

$$p(x_t|y_{1:T}) \propto \bar{p}(x_t|y_{t+1:T}) \int p(x_t|x_{t-1}) p(x_{t-1}|y_{1:t-1}) dx_{t-1}$$

$\{\overleftrightarrow{x}_t^{(1:N)}, \overleftrightarrow{w}_t^{(1:N)}\}$:= Smoother Particle Set
$\{\vec{x}_t^{(1:N)}, \vec{w}_t^{(1:N)}\}$:= Forward Filter Particle Set
$\{\overleftarrow{x}_t^{(1:N)}, \overleftarrow{w}_t^{(1:N)}\}$:= Backward Filter Particle Set

Particle based algorithm:

$$\overleftrightarrow{w}_t^{(j)} \propto \overleftarrow{w}_t^{(j)} \sum_{i=1}^{N} \vec{w}_t^{(i)} p(\overleftarrow{x}_t^{(j)}|\vec{x}_t^{(i)}) \quad \text{with } \overleftrightarrow{x}_t^{(i)} = \overleftarrow{x}_t^{(i)}$$

Particle Smoothing Algorithm:
1. Run backwards in time particle filter
2. Re-weight backwards particles using forward filter

4. Mixture Density Particle Smoother (MDPS)

Starting from Two Filter Smoother:

$$p(x_t|y_{1:T}) \propto \underbrace{p(x_t|y_{1:t})}_{\text{Forward Filtering}} \cdot \underbrace{\bar{p}(x_t|y_{t+1:T})}_{\text{Backward Filtering}}$$

We can derive:

$$p(x_t|y_{1:T}) \propto l(x_t|y_t)\, \underbrace{p(x_t|y_{1:t-1})}_{\text{Forward Filtering}} \cdot \underbrace{\bar{p}(x_t|y_{t+1:T})}_{\text{Backward Filtering}}$$

Where:

$$p(x_t|y_{1:t-1}) = \sum_{i=1}^{N} \vec{w}_t^{(i)} \cdot K(x; \vec{x}_t^{(i)}, \vec{\beta}) \qquad p(x_t|y_{t+1:T}) = \sum_{i=1}^{N} \overleftarrow{w}_t^{(i)} \cdot K(x; \overleftarrow{x}_t^{(i)}, \overleftarrow{\beta})$$

(Forward Filtering Posterior) *(Backward Filtering Posterior)*

Set smoothed particles:

$$\overleftrightarrow{x}_t^{(i)} \sim q(x_t) = \frac{1}{2} p(x_t|y_{1:t-1}) + \frac{1}{2} p(x_t|y_{t+1:T}) \qquad i = 1,\ldots,M$$

Compute weights:

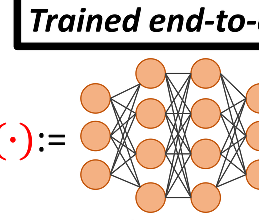
$$\overleftrightarrow{w}_t^{(i)} \propto \frac{l(\overleftrightarrow{x}_t^{(i)}|y_t)\; p(\overleftrightarrow{x}_t^{(i)}|y_{1:t-1}) \cdot \bar{p}(\overleftrightarrow{x}_t^{(i)}|y_{t+1:T})}{q(\overleftrightarrow{x}_t^{(i)})} \qquad \sum_{i=1}^{N} \overleftrightarrow{w}_t^{(i)} = 1$$

Parameterize via a Neural Network:

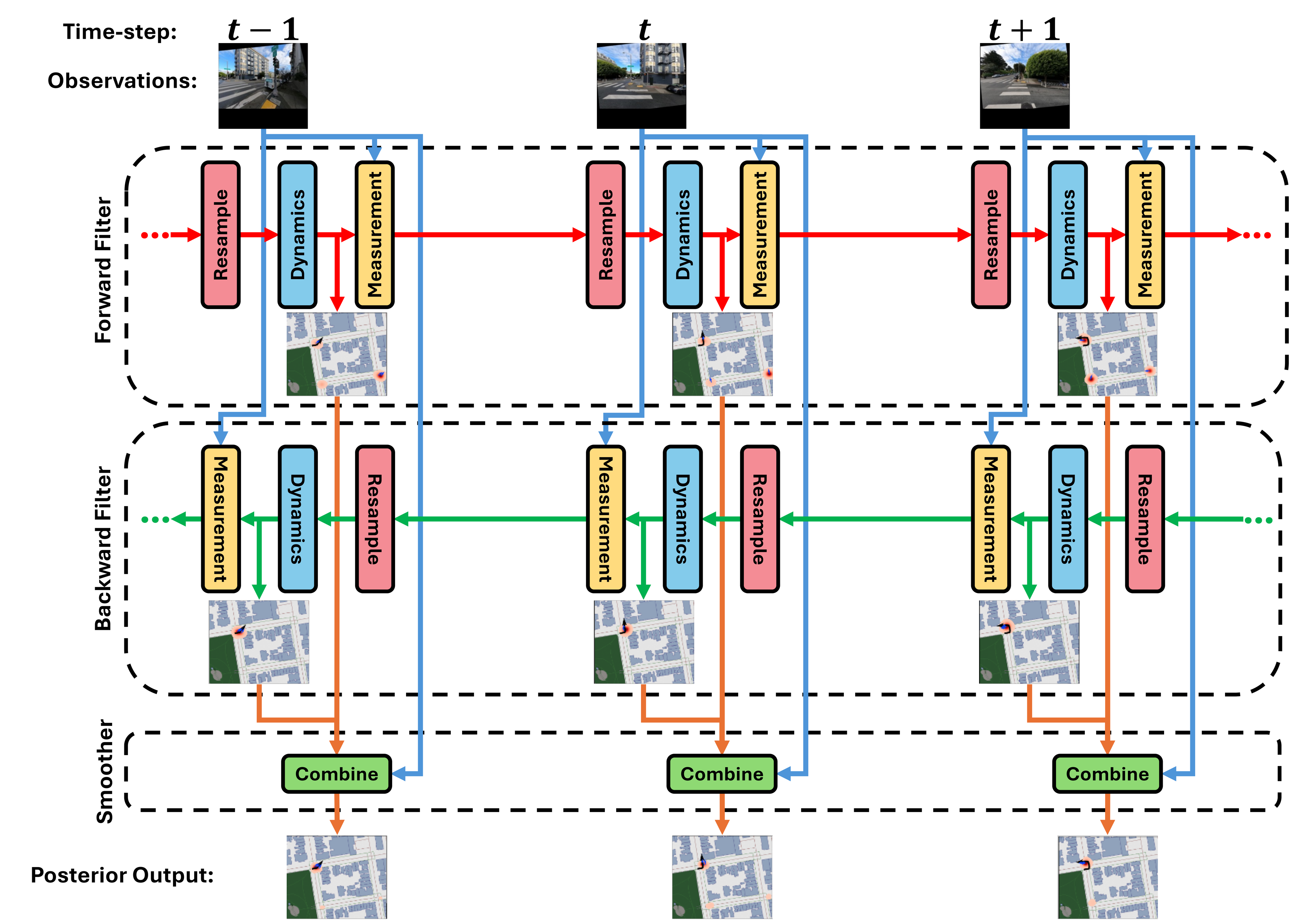
$$\overleftrightarrow{w}_t^{(i)} \propto \frac{l\left(\overleftrightarrow{x}_t^{(i)}; y_t, p(\overleftrightarrow{x}_t^{(i)}|y_{1:t-1}), \bar{p}(\overleftrightarrow{x}_t^{(i)}|y_{t+1:T})\right)}{q(\overleftrightarrow{x}_t^{(i)})}$$

Trained end-to-end

$l(\cdot)$:=



Neural Network



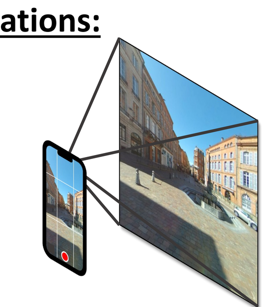
5. Results

City Scale Global Localization Task

Task: Estimate 3D state (position and heading) of a subject as it moves through a real-world city-scale environment.

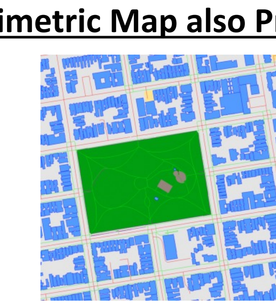
Actions: Noisy odometry

Observations:



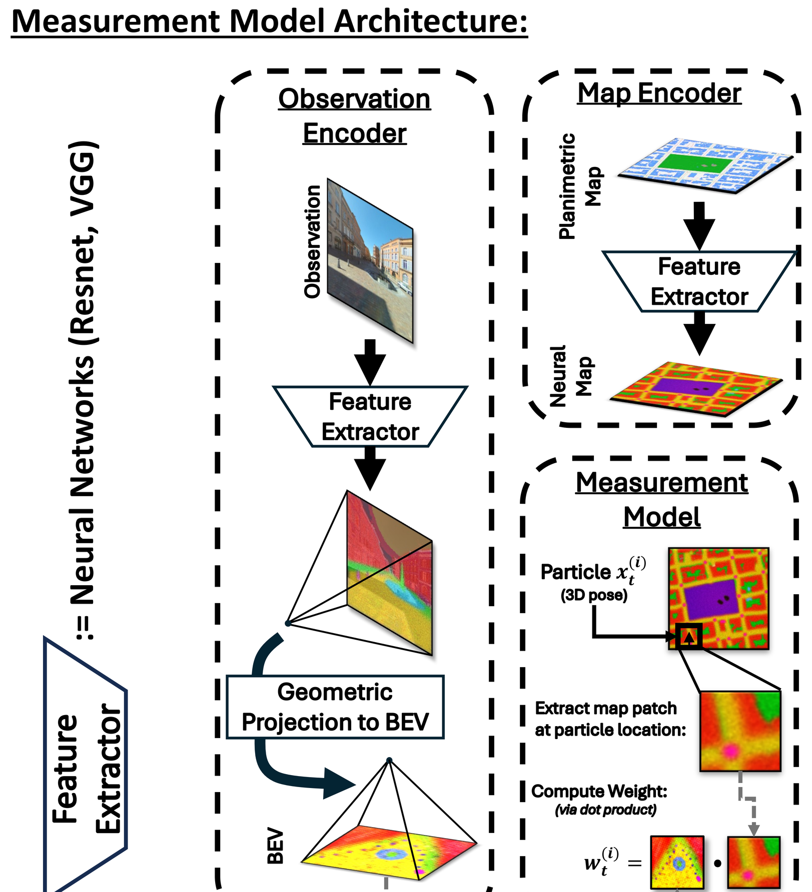
(First person RGB images)

Planimetric Map also Provided:



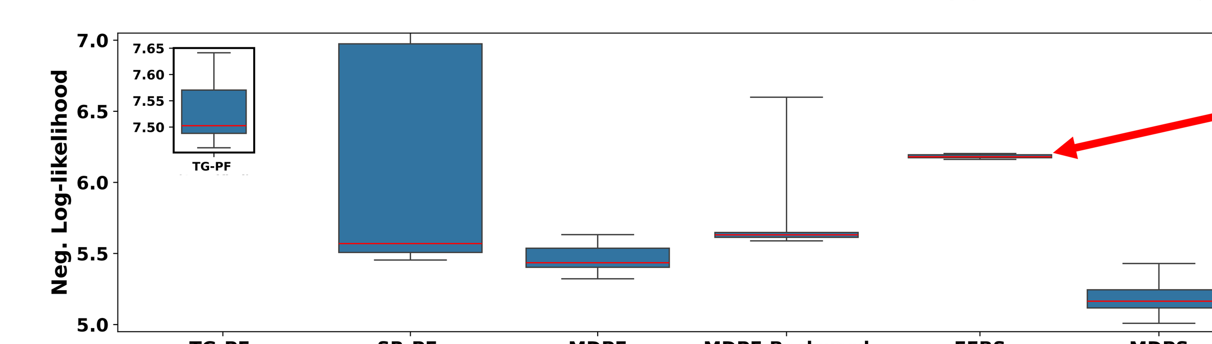
(OpenStreetMap)

Measurement Model Architecture:



Bearings Only Tracking Task

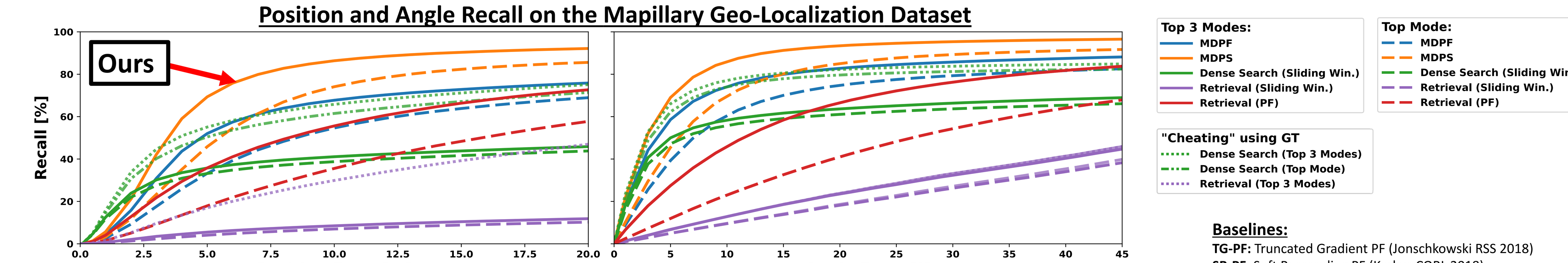
Observations: $y_t \sim \alpha \cdot \text{Uniform}(-\pi,\pi) + (1-\alpha) \cdot \text{VonMises}(\psi(x_t),\kappa)$ $\psi(x_t)$:= true bearing



Classical Particle Smoother method:
- State Dynamics -> Learned outside of smoother algorithm
- Used ***True*** Observation Likelihood

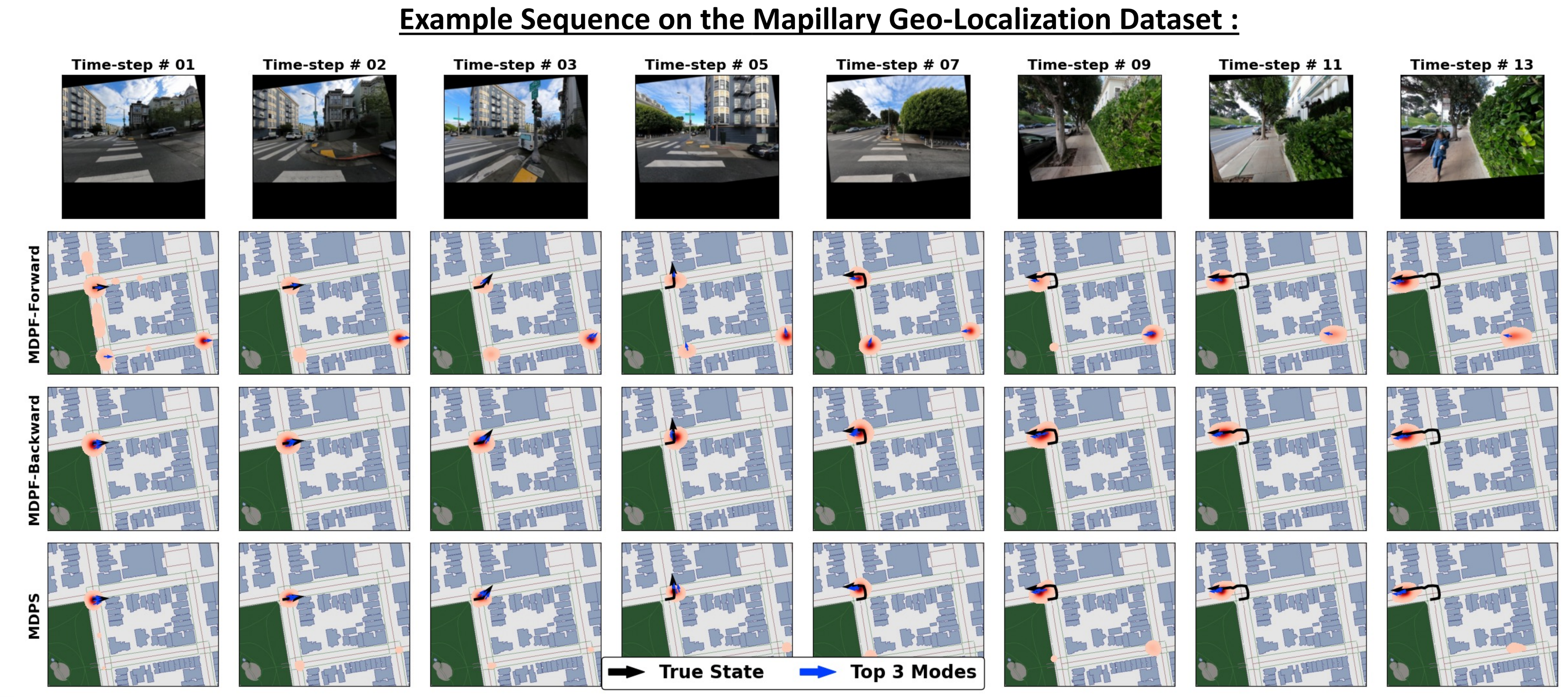
No Existing Differentiable Particle Smoothers!

Position and Angle Recall on the Mapillary Geo-Localization Dataset

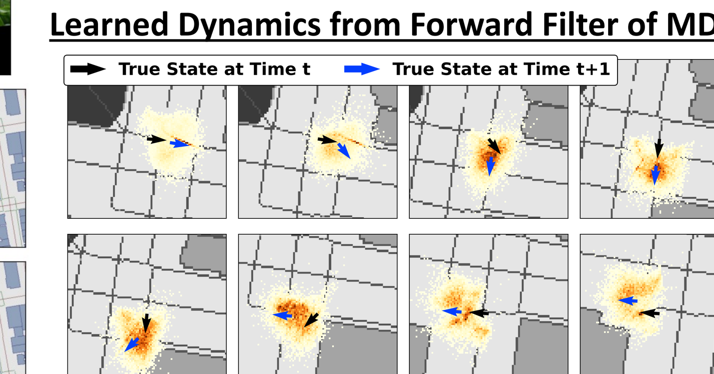


Baselines:
- **TG-PF:** Truncated Gradient PF (Jonschkowski RSS 2018)
- **SR-PF:** Soft Resampling PF (Karkus CORL 2018)
- **MDPF:** Mixture Density PF (Younis NeurIPS 2023)
- **Dense Search:** OrienterNet (Sarlin CVPR 2023)
- **Retrieval (PF):** PF with Retrieval method measurement model (Zhou IROS 2021)
- **Retrieval (Sliding Win.):** Retrieval method lookup (Samano ECCV 2021, implemented as in Sarlin CVPR 2023)

Example Sequence on the Mapillary Geo-Localization Dataset :



Learned Dynamics from Forward Filter of MDPS



This research supported in part by:
- NSF Robust Intelligence Award No.IIS-1816365
- ONR Award No.N00014-23-1-2712